

### Details of Thomson's $e/m$ Calculations

To calculate  $e/m$  it was first necessary to determine the velocity of the electrons in the cathode rays. This was done by deflecting the beam with a magnetic field,  $H$ , and then applying an opposing electrical field,  $E$ , just strong enough to restore a straight-line path. (Note: Vector direction of  $H$  is N→S.) The forces exerted on the traveling electrons are given by

$$F_{\text{mag}} = Hev \quad \text{and} \quad F_{\text{elec}} = Ee$$

where  $e$  is the charge on the electron and  $v$  is its velocity. With both fields adjusted to give straight-line motion,  $F_{\text{mag}} = F_{\text{elec}}$ , so

$$\begin{aligned} Hev &= Ee \\ \Rightarrow v &= E/H \end{aligned} \tag{1}$$

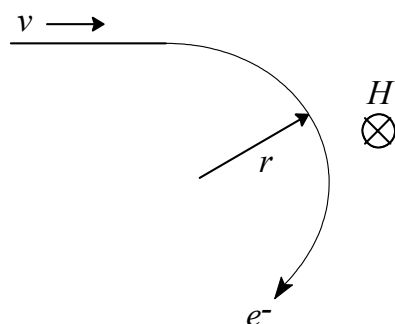
By determining the necessary applied  $E$  and  $H$  fields to achieve a straight-line beam, the velocity  $v$  could be determined from equation (1). Thomson observed that the velocity only depended on the voltage across the cathode and anode.

With  $v$  known as a function of applied tube voltage, Thomson was able to determine the ratio  $e/m$  by two methods. We will only consider the more direct of these. With the magnetic field applied alone, the beam is bent by a force  $F_{\text{mag}} = Hev$ . But any force is mass times acceleration,  $F = ma$ , so

$$\begin{aligned} F_{\text{mag}} &= Hev = ma \\ \Rightarrow e/m &= a/Hv \end{aligned} \tag{2}$$

At this point both  $H$  and  $v$  were known, so a means of getting the acceleration,  $a$ , was required to solve equation (2).

When the beam of negatively charged electrons is deflected by the applied magnetic field,  $H$ , it begins to assume a circular trajectory with a radius of curvature,  $r$ .



This radius can be determined by observing the point at which the curved beam strikes the fluorescent screen at the end of the tube.

With  $r$  now known, the acceleration can be calculated by using the fundamental equation for the acceleration of a body moving in a circular trajectory:

$$a = v^2/r \quad (3)$$

Substituting this value of  $a$  from equation (3) into the  $e/m$  equation (2) gives

$$e/m = \frac{v^2/r}{Hv} = v/Hr \quad (4)$$

With  $v$ ,  $H$ , and  $r$  known,  $e/m$  could be calculated by equation (4).