# Details of Milliken's Calculations

A small oil drop falling through air does not continuously accelerate. Instead, because of its drag against the relatively viscous air, the drop assumes a constant velocity, given by

$$v = mg/6\pi r\eta \tag{1}$$

where $m$ is the mass of the drop, $g$ is the acceleration of gravity, $r$ is the drop's radius, and $\eta$ (Greek eta) is the viscosity of the air. Assuming that the drop is a sphere, its volume is $V = (4/3)\pi r^3$, and its density is therefore

$$d = m/V = 3m/4\pi r^3 \tag{2}$$

This density must be the same as the density of the oil in the atomizer, used to produce the spray of drops. The density of the oil in the atomizer could be determined easily from a bulk sample. Combining equations (1) and (2), expressions for $r$ and $m$ can be obtained:

$$r = 3\sqrt{\eta v/2dg} \tag{3}$$

$$m = 9\pi\sqrt{2\eta^3 v^3/dg^3} \tag{4}$$

With the density of the oil ($d$), the viscosity of the air ($\eta$), and the acceleration of gravity ($g$) all known, observing the velocity of fall of the drop ($v$) in the absence of an electrical field gave the necessary data to use in equations (3) and (4) to calculate the radius ($r$) and mass ($m$) of the drop.

Once the data to calculate $r$ and $m$ were obtained, the plates were charged with an electric field, $E$, which acted on the negative charge of the drop, $q$, to give a retarded velocity, $v_e$, given by

$$v_e = (qE - mg)/6\pi r\eta \tag{5}$$

With everything but $q$ known in equation (5), the charge on a drop could be calculated from the observed retarded velocity, $v_e$. The charges on individual drops varied from drop to drop, depending on how many electrons they had acquired. But in every case the charge was a whole-number multiple of $-1.60 \times 10^{-19}$ C, which Milliken took to be the charge on a single electron, the unit negative charge.