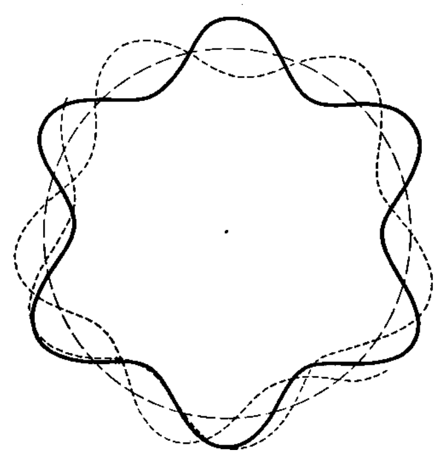
How the de Broglie Equation Seemed to Justify Bohr's Quantum Hypothesis

The de Broglie equation, $\lambda = h/mv$, which asserts wave-particle duality for matter, such as an electron, on the one hand seemed at odds with the particle-only view of Bohr's model. On the other hand, it did provide a justification for the seemingly arbitrary quantum hypothesis of Bohr's model, the assertion that the electron's angular momentum should be quantized in units of $h/2\pi$; i.e., $mvr = nh/2\pi$. To understand this, image the electron as a wave train processing about one of Bohr's circular orbits. To establish a standing wave, each time the wave train came around the circular orbit it would have to end at the same point it started, as shown by the heavy line in diagram below. In other words, the wavelength of the electron, λ, had to be such that a whole number of waves fit around the circle of the orbit.



If the wavelength did not fit this requirement, waves would overlap out of phase (fine dotted line in the diagram) and destroy one another by interference. Now, by geometry the circumference of the circular orbit is $2\pi r$, where r is the radius. Thus, the requirement that an integer number of waves fit around this circumference means that the circumference must equal a whole number of wavelengths. This can be expressed by the equation

$$n\lambda = 2\pi r \qquad n = 1, 2, 3, \ldots \tag{1}$$

But according to the de Broglie equation, $\lambda = h/mv$. Substituting this into equation (1) gives

$$\frac{nh}{mv} = 2\pi r$$

which can be rearranged as

$$mvr = \frac{nh}{2\pi} \tag{2}$$

Equation (2) is the quantum hypothesis that Bohr had introduced onto his classical model, seemingly arbitrarily. With the de Broglie equation, this condition arises naturally as a requirement to avoid interference that would preclude establishing the fixed states of the Bohr model.