Relationships from the Great Orthogonality Theorem

1. The sum of the squares of the dimensions of all the irreducible representations is equal to the order of the group:

$$\sum_i d_i^2 = h = \sum_i [\chi_i(E)]^2$$

C_{3v}	E	$2C_3$	$3\sigma_v$	$h = 6$
A_1	1	1	1	$d_i^2 = 1$
A_2	1	1	-1	$d_i^2 = 1$
E	2	-1	0	$d_i^2 = 4$
				$\Sigma d_i^2 = 6$

2. The number of irreducible representations of a group is equal to the number of classes.

3. In a given representation (irreducible or reducible) the characters for all operations belonging to the same class are the same.

Relationships from the
Great Orthogonality Theorem

4. The sum of the squares of the characters in any irreducible representation equals the order of the group:

$$\sum_{R} [\chi_i(R)]^2 = h = \sum_{R_c} g_c[\chi_i(R_c)]^2$$

C_{3v}	E	$2C_3$	$3\sigma_v$	$h = 6$
A_1	1	1	1	(1)(1) + (2)(1) + (3)(1) = 6
A_2	1	1	-1	(1)(1) + (2)(1) + (3)(1) = 6
E	2	-1	0	(1)(4) + (2)(1) + (3)(0) = 6

5. Any two different irreducible representations are orthogonal:

$$\sum_{R_c} g_c\chi_i(R_c)\chi_j(R_c) = 0$$

C_{3v}	E	$2C_3$	$3\sigma_v$	
A_2	1	1	-1	
E	2	-1	0	$\Sigma g_c\chi_i\chi_j$
$g_c\chi_i\chi_j$	(1)(1)(2) = 2	(2)(1)(-1) = -2	(3)(-1)(0) = 0	2 – 2 + 0 = 0

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Combining points 4 and 5:

$$\sum_{R_c} g_c \chi_i(R_c)\chi_j(R_c) = h\delta_{ij}$$

Kronecker delta function, δ_{ij}:

$$\delta_{ij} = 0 \text{ if } i \neq j$$

$$\delta_{ij} = 1 \text{ if } i = j$$