

Symmetry Point Groups

- ☞ The complete collection of symmetry operations (*not* symmetry elements) satisfies the requirements of a mathematical group.
 - ✓ The symmetry operations are the *elements* of a group.
 - ✓ The total number of symmetry operations comprising the group is the *order of the group*, h .
- ☞ The group formed by the operations of CBr_2Cl_2 is named C_{2v} .

C_{2v}	E	C_2	σ_v	σ_v'
E	E	C_2	σ_v	σ_v'
C_2	C_2	E	σ_v'	σ_v
σ_v	σ_v	σ_v'	E	C_2
σ_v'	σ_v'	σ_v	C_2	E

Group Requirements

- ① **Closure** If A and B are in the group G , and $AB = X$, then X is also in G .
- ✓ All groups have a self-contained multiplication table, whose products are members of the group.
- ② **Identity** In any group G , there is an element E , such that
- $$EX = XE = X$$
- ✓ The symmetry operation of identity is this group element.
- ③ **Associativity** If A , B , C , and X are in G , then
- $$C(BA) = X = (CB)A$$
- ✓ But commutation is not general (e.g., $S_4\sigma_v \neq \sigma_v S_4$).
 - ✓ Groups in which all elements *do* commute are called **Abelian** (e.g., C_{2v}).
- ④ **Reciprocity** In any group G , every element A has an inverse A^{-1} , such that $AA^{-1} = A^{-1}A = E$.
- ✓ An element may be its own inverse (e.g., all operations of C_{2v}).

Subgroups

☞ Within all groups (except the trivial asymmetric group, $C_1 = \{E\}$), there are smaller collections of elements, called **subgroups**, which also obey the criteria for a group.

☞ The order of any subgroup, g , relative to the order of its parent group, h , must be

$$h/g = n \quad n = 1, 2, \dots$$

✓ Not every allowed value of g is always represented among a group's subgroups.

Subgroups of C_{2v} ($h/g = 4/g = 1, 2$)

Group Label	Operations (Group Elements)
C_1	E
C_2	E, C_2
C_s	E, σ_v or E, σ_v'