Symmetry Operations and Elements

• The goal for this section of the course is to understand how symmetry arguments can be applied to solve physical problems of chemical interest.

• To achieve this goal we must identify and catalogue the complete symmetry of a system and subsequently employ the mathematics of groups to simplify and solve the physical problem in question.

• A **symmetry element is an imaginary geometrical construct** about which a symmetry operation is performed.

• A **symmetry operation is a movement of an object about a symmetry element** such that the object's orientation and position before and after the operation are *indistinguishable*.

• A symmetry operation carries every point in the object into an *equivalent point or the identical point*. 
Point Group Symmetry

- All symmetry elements of a molecule pass through a central point within the molecule.
- The more symmetry operations a molecule has, the higher its symmetry is.
- Regardless of how many or few symmetry operations a molecule possesses, all are examples of one of five types.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Element</th>
<th>Element Construct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity, $E$</td>
<td>The object</td>
<td>N/A</td>
</tr>
<tr>
<td>Proper rotation, $C_n$</td>
<td>Proper axis, Rotation axis</td>
<td>line</td>
</tr>
<tr>
<td>Reflection, $\sigma$</td>
<td>Mirror plane, Reflection plane</td>
<td>plane</td>
</tr>
<tr>
<td>Inversion, $i$</td>
<td>Inversion center, Center of symmetry</td>
<td>point</td>
</tr>
<tr>
<td>Rotation-reflection</td>
<td>Improper axis, alternating axis</td>
<td>line</td>
</tr>
<tr>
<td>Improper rotation, $S_n$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Point Group Symmetry

- If a molecule has rotational symmetry, rotation by $2\pi/n = 360°/n$ brings the object into an equivalent position.
- The value of $n$ is the order of an $n$-fold rotation.
- If the molecule has one or more rotational axes, the one with the highest value of $n$ is the principal axis of rotation.

Successive $C_4$ clockwise rotations of a planar MX$_4$ molecule about an axis perpendicular to the plane of the molecule ($X_A = X_B = X_C = X_D$).
The $C_2'$ and $C_2''$ axes of a planar MX$_4$ molecule.
General Relationships for $C_n$

\[ C_n^n = E \]
\[ C_{2n}^n = C_2 \quad (n = 2, 4, 6, 8... \text{etc.}) \]
\[ C_n^m = C_{m/n} \quad (m/n = 2, 3, 4, 5... \text{etc.}) \]
\[ C_n^{n-1} = C^{-n}_n \]
\[ C_n^{n+m} = C_n^m \quad (m < n) \]

• Every $n$-fold rotational axis has $n-1$ associated operations (excluding $C_n^n = E$).
Reflection, $\sigma$

Two points, equidistant from a mirror plane $\sigma$, related by reflection.

- For a point $(x, y, z)$, reflection across a mirror plane $\sigma_{xy}$ takes the point into $(x, y, -z)$.
- Each mirror plane has only one operation associated with it
  i.e., $\sigma^2 = E$. 
Horizontal, Vertical, and Dihedral Mirror Planes

Mirror planes of a square planar molecule MX$_4$. 
Inversion, \( i \)

- If inversion symmetry exists, for every point \((x,y,z)\) there is an equivalent point \((-x,-y,-z)\).
- Each inversion center has only one operation associated with it, since \(i^2 = E\).

Effect of inversion \((i)\) on an octahedral \(MX_6\) molecule \((X_A = X_B = X_C = X_D = X_E = X_F)\).
Inversion, $i$

- Ethane in the staggered configuration. The inversion center is at the midpoint along the C-C bond. Hydrogen atoms related by inversion are connected by dotted lines, which intersect at the inversion center. The two carbon atoms are also related by inversion.
Rotation-Reflection (Improper Rotation), $S_n$

- $S_n$ exists if the movements $C_n$ followed by $\sigma_h$ (or vice versa) bring the object to an equivalent position.
- If both $C_n$ and $\sigma_h$ exist, then $S_n$ must exist.
- Example: $S_4$ collinear with $C_4$ in planar MX$_4$.
- Neither $C_n$ nor $\sigma_h$ need exist for $S_n$ to exist.
- Example: $S_4$ collinear with $C_2$ in tetrahedral MX$_4$.

$S_4$ improper rotation of a tetrahedral MX$_4$ molecule ($X_A = X_B = X_C = X_D$). The improper axis is perpendicular to the page. Rotation is arbitrarily taken in a clockwise direction. Note that neither $C_4$ nor $\sigma_h$ are genuine symmetry operations of tetrahedral MX$_4$. 
Successive $S_4$ operations on a tetrahedral MX$_4$ molecule ($X_A = X_B = X_C = X_D$). Rotations are clockwise, except $S_4^{-1}$, which is equivalent to the clockwise operation $S_4^3$. 
A tetrahedral MX₄ molecule inscribed in a cube. A C₂ axis, collinear with an S₄ axis, passes through the centers of each pair of opposite cube faces and through the center of the molecule.
Non-Genuine $S_n$ Operations:

- $S_1 = \sigma$
- $S_2 = i$
- The lowest order $S_n$ operation that is not a simpler operation is $S_3$. 
General Relations of Improper Axes

• Equivalences of successive $S_n$ operations:
  
  ➢ If $n$ is even, $S_n^n = E$

  ➢ If $n$ is odd, $S_n^n = \sigma$ and $S_n^{2n} = E$

  ➢ If $m$ is even, $S_n^m = C_{n/m}$ when $m < n$ and $S_n^m = C_n^{m-n}$ when $m > n$

  ➢ If $S_n$ with even $n$ exists, then $C_{n/2}$ exists

  ➢ If $S_n$ with odd $n$ exists, then both $C_n$ and $\sigma$ perpendicular to $C_n$ exist.
Examples

• Find all symmetry elements and operations in the following:
Defining the Coordinate System

• Molecules are conventionally oriented relative to a right-hand Cartesian coordinate system:

• The following conventions of axis orientation are usually observed:

  1. The origin of the coordinate system is located at the central atom or the center of the molecule.

  2. The z axis is collinear with the highest-order rotational axis (the principal axis). If there are several highest order rotational axes, z is usually taken as the axis passing through the greatest number of atoms.

However, for a tetrahedral molecule, the x, y, and z axes are defined as collinear with the three $C_2$ axes (collinear with the three $S_4$ axes).
Defining the Coordinate System (contd.)

3. For planar molecules, if the $z$ axis as defined above is perpendicular to the molecular plane, the $x$ axis lies in the plane of the molecule and passes through the greatest number of atoms.

If the $z$ axis lies in the plane of the molecule, then the $x$ axis stands perpendicular to the plane.
4. For non-planar molecules, once the $z$ axis has been defined, the $x$ axis is usually chosen so that the $xz$ plane contains as many atoms as possible. If there are two or more such planes containing identical sets of atoms, any one may be taken as the $xz$ plane.

Where a decision about the orientation of the $x$ axis cannot be made on this basis, the distinction between $x$ and $y$ is usually not important or is not generally fixed by convention.
Combining Symmetry Operations (Multiplication)

- Multiplication of symmetry operations is the successive performance of two or more operations to achieve an orientation that could be reached by a single operation.

- The order in which successive different symmetry operations are performed can affect the result.

- *Multiplication of symmetry operations is not in general commutative, although certain combinations may be.*

- In writing multiplications of symmetry operation we use a "right-to-left" notation:
  - $BA = X$ "Doing A then B has the same result as the operation $X$.
  - We cannot assume that reversing the order will have the same result.
  - It may be that either $BA \neq AB$ or $BA = AB$.

- *Multiplication is associative:*

  $$C(BA) = (CB)A$$
The order of performing $S_4$ and $\sigma_v$, shown here for a tetrahedral MX$_4$ molecule, affects the result. The final positions in each case are not the same, but they are related to each other by $C_2$. 
Multiplication Tables

- All possible binary combinations of symmetry operations can be summarized in a multiplication table.

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>E</td>
<td>E</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>C</td>
<td>E</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>E</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

- Combination order is "top" then "side"; e.g.,

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>EE = E</td>
<td>EA = A</td>
<td>EB = B</td>
<td>EC = C</td>
</tr>
<tr>
<td>A</td>
<td>AE = A</td>
<td>AA = B</td>
<td>AB = C</td>
<td>AC = E</td>
</tr>
<tr>
<td>B</td>
<td>BE = B</td>
<td>BA = C</td>
<td>BB = E</td>
<td>BC = A</td>
</tr>
<tr>
<td>C</td>
<td>CE = C</td>
<td>CA = E</td>
<td>CB = A</td>
<td>CC = B</td>
</tr>
</tbody>
</table>
• Symmetry elements of CBr₂Cl₂
Matrix Notation of the Effects of the Operations

\[
\begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} \times \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} = \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} \times \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} = \begin{bmatrix}
\text{Br}_a \\
\text{Br}_a \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} \times \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} = \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} \times \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix} = \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b
\end{bmatrix}
\]

- Multiplication .............
# Multiplication Table for the Operations of CBr$_2$Cl$_2$

- **Step 1: Combinations with identity.**

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$C_2$</th>
<th>$\sigma_v$</th>
<th>$\sigma_v'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$E$</td>
<td>$C_2$</td>
<td>$\sigma_v$</td>
<td>$\sigma_v'$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$C_2$</td>
<td>$C_2$</td>
<td>$\sigma_v$</td>
<td>$\sigma_v'$</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>$\sigma_v$</td>
<td>$\sigma_v$</td>
<td>$\sigma_v'$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_v'$</td>
<td>$\sigma_v'$</td>
<td>$\sigma_v'$</td>
<td>$\sigma_v'$</td>
<td></td>
</tr>
</tbody>
</table>

- **Step 2: Binary self-combinations.**

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$C_2$</th>
<th>$\sigma_v$</th>
<th>$\sigma_v'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$E$</td>
<td>$C_2$</td>
<td>$\sigma_v$</td>
<td>$\sigma_v'$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$C_2$</td>
<td>$E$</td>
<td>$\sigma_v$</td>
<td>$E$</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>$\sigma_v$</td>
<td>$E$</td>
<td>$\sigma_v'$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_v'$</td>
<td>$\sigma_v'$</td>
<td>$E$</td>
<td>$\sigma_v'$</td>
<td></td>
</tr>
</tbody>
</table>
Step 3: Mixed binary combinations.

\[ C_2 \sigma_v = ? \]

\[
\begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix} \times \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix} = \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix} \times \begin{bmatrix}
\text{Br}_v' \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix} = \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix}
\]

This result is the same as that achieved by \( \sigma_v' \) alone:

\[
\begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix} \times \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix} = \begin{bmatrix}
\text{Br}_a \\
\text{Br}_b \\
\text{Cl}_a \\
\text{Cl}_b \\
\end{bmatrix}
\]

\[ C_2 \sigma_v = \sigma_v' \]
Complete Multiplication Table

<table>
<thead>
<tr>
<th></th>
<th>$E$</th>
<th>$C_2$</th>
<th>$\sigma_v$</th>
<th>$\sigma_v'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$E$</td>
<td>$C_2$</td>
<td>$\sigma_v$</td>
<td>$\sigma_v'$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$C_2$</td>
<td>$E$</td>
<td>$\sigma_v'$</td>
<td>$\sigma_v$</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>$\sigma_v$</td>
<td>$\sigma_v'$</td>
<td>$E$</td>
<td>$C_2$</td>
</tr>
<tr>
<td>$\sigma_v'$</td>
<td>$\sigma_v'$</td>
<td>$\sigma_v$</td>
<td>$C_2$</td>
<td>$E$</td>
</tr>
</tbody>
</table>

- **General Results:**
  - The first row of results duplicates the list of operations in the header row.
  - The first column of results duplicates the list of operations in the label column.
  - Every row shows every operation once and only once.
  - Every column shows every operation once and only once.
  - The order of resultant operations in every row is different from any other row.
  - The order of resultant operations in every column is different from any other column.
Symmetry Point Groups

- The complete collection of symmetry operations (not symmetry elements) satisfies the requirements of a mathematical group.
- The symmetry operations are the elements of a group.
- The total number of symmetry operations comprising the group is the order of the group, $h$.
- The group formed by the operations of CBr$_2$Cl$_2$ is named C$_{2v}$.

\[
\begin{array}{c|cccc}
  & E & C_2 & \sigma_v & \sigma_v' \\
\hline
  E & E & C_2 & \sigma_v & \sigma_v' \\
  C_2 & C_2 & E & \sigma_v' & \sigma_v \\
  \sigma_v & \sigma_v & \sigma_v' & E & C_2 \\
  \sigma_v' & \sigma_v' & \sigma_v & C_2 & E \\
\end{array}
\]
Group Requirements

- **Closure:** If $A$ and $B$ are in the group $G$, and $AB = X$, then $X$ is also in $G$.

  All groups have a self-contained multiplication table, whose products are members of the group.

- **Identity:** In any group $G$, there is an element $E$, such that

  $$EX =XE = X$$

  The symmetry operation of identity is this group element.

- **Associativity:** If $A$, $B$, $C$, and $X$ are in $G$, then

  $$C(BA) = X = (CB)A$$

  But commutation is not general (e.g., $S_4 \sigma_v \neq \sigma_v S_4$).

  Groups in which all elements *do* commute are called Abelian (e.g., $C_{2v}$).

- **Reciprocal:** In any group $G$, every element $A$ has an inverse $A^{-1}$, such that

  $$AA^{-1} = A^{-1}A = E$$

  An element may be its own inverse (e.g., all operations of $C_{2v}$).
Subgroups

• *Within all groups there are smaller collections of elements, called subgroups, which also obey the criteria for a group.*

  *note: one exception is the trivial asymmetric group, \( C_1 = \{E\} \)*

The order of any subgroup, \( g \), relative to the order of its parent group, \( h \), must be

\[
h / g = n \quad (n = 1, 2, 3, 4, \ldots \text{etc.})
\]

Not every allowed value of \( g \) is always represented among a group’s subgroups.

Subgroups of \( C_{2v} \) (\( h / g = 4 / g = 1, 2 \))

<table>
<thead>
<tr>
<th>Group Label</th>
<th>Operations (Group Elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>( E )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( E, C_2 )</td>
</tr>
<tr>
<td>( C_s )</td>
<td>( E, \sigma_v ) or ( E, \sigma_v' )</td>
</tr>
</tbody>
</table>
Point Groups of Molecules

- Chemists in general and spectroscopists in particular use the Schönflies notation; crystallographers use the Hermann-Mauguin notation.

  e.g.,

<table>
<thead>
<tr>
<th>Schönflies</th>
<th>Hermann-Mauguin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>1</td>
</tr>
<tr>
<td>$C_s$</td>
<td>$m$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>2</td>
</tr>
<tr>
<td>$C_{2v}$</td>
<td>$mm$</td>
</tr>
<tr>
<td>$D_2$</td>
<td>222</td>
</tr>
<tr>
<td>$D_{3h}$</td>
<td>$(3/m)mm$</td>
</tr>
</tbody>
</table>
Common Point Groups and Their Principal Operations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Nonrotational Groups</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td></td>
<td>$E$ (asymmetric)</td>
</tr>
<tr>
<td>$C_s$</td>
<td></td>
<td>$E$, $\sigma_h$</td>
</tr>
<tr>
<td>$C_i$</td>
<td></td>
<td>$E$, $i$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Single-axis Groups</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_n$</td>
<td></td>
<td>$E$, $C_n$, $\ldots$, $C_n^{n-1}$</td>
</tr>
<tr>
<td>$C_{nm}$</td>
<td></td>
<td>$E$, $C_n$, $\ldots$, $C_n^{n-1}$, $n\sigma_v$ ($n/2$ $\sigma_d$ and $n/2$ $\sigma_d$ if $n$ even)</td>
</tr>
<tr>
<td>$C_{nh}$</td>
<td></td>
<td>$E$, $C_n$, $\ldots$, $C_n^{n-1}$, $\sigma_h$</td>
</tr>
<tr>
<td>$S_{2n}$</td>
<td></td>
<td>$E$, $S_{2n}$, $\ldots$, $S_{2n}^{2n-1}$</td>
</tr>
<tr>
<td>$C_{\infty v}$</td>
<td></td>
<td>$E$, $C_{\infty}$, $\infty\sigma_v$ (noncentrosymmetric linear)</td>
</tr>
</tbody>
</table>
Common Point Groups and Their Principal Operations

### Dihedral Groups

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Operations</th>
<th>((n = 2, 3, \ldots, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_n)</td>
<td>(E, C_n, \ldots, C_n^{n-1}, nC_2(\perp C_n))</td>
<td></td>
</tr>
<tr>
<td>(D_{nd})</td>
<td>(E, C_n, \ldots, C_n^{n-1}, S_{2n}, \ldots, S_{2n}^{2n-1}, nC_2(\perp C_n), n\sigma_d)</td>
<td></td>
</tr>
<tr>
<td>(D_{nh})</td>
<td>(E, C_n, \ldots, C_n^{n-1}, nC_2(\perp C_n), \sigma_h, n\sigma_v)</td>
<td></td>
</tr>
<tr>
<td>(D_{oh})</td>
<td>(E, C_{\infty}, \infty C_2(\perp C_{\infty}), \infty\sigma_v, \infty\sigma_d) (i) (centrosymmetric linear)</td>
<td></td>
</tr>
</tbody>
</table>

### Cubic Groups

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_d)</td>
<td>(E, 4C_3, 4C_3^2, 3C_2, 3S_4, 3S_4^3, 6\sigma_d) () (tetrahedron)</td>
</tr>
<tr>
<td>(O_h)</td>
<td>(E, 4C_3, 4C_3^2, 6C_2, 3C_4, 3C_4^3, 3C_2 (= C_4^2), i, 3S_4, 3S_4^3, 4S_6, 4S_6^5, 3\sigma_h, 6\sigma_d) () (octahedron)</td>
</tr>
<tr>
<td>(I_h)</td>
<td>(E, 6C_5, 6C_5^2, 6C_5^3, 6C_5^4, 10C_3, 10C_3^2, 15C_2, i, 6S_{10}, 6S_{10}^3, 6S_{10}^7, 6S_{10}^6, 10S_6, 10S_6^5, 15\sigma) () (icosahedron, dodecahedron)</td>
</tr>
</tbody>
</table>
Cyclic Groups

- A cyclic group of order $h$ is generated by taking a single element $X$ through all its powers to $X_h = E$.

$$G = \{ X, X_2, \ldots, X_h = E \}$$

- All cyclic groups are Abelian.

- The $C_n$ and $S_{2n}$ groups are cyclic groups; e.g.,

$$C_4 = \{ C_4, C_4^2, C_4^3, E \}$$

$$S_4 = \{ S_4, C_2, S_4^3, E \}$$

- The multiplication tables of cyclic groups "scroll" from row to row and column to column: e.g.,

$$\begin{array}{c|cccc}
C_4 & E & C_4 & C_2 & C_4^3 \\
\hline
E & E & C_4 & C_2 & C_4^3 \\
C_4 & C_4 & C_2 & C_4^3 & E \\
C_2 & C_2 & C_4^3 & E & C_4 \\
C_4^3 & C_4^3 & E & C_4 & C_2 \\
\end{array}$$
Examples of molecules with various point group symmetries
Flow chart for systematically determining the point group of a molecule.
Examples for point group classification.
Representations of the three conformations of ethane as two triangles separated along the $C_3$ axis. The corresponding Newman projections are shown on the right.