

Quantum Mechanics
CH 602
Fall 2006
Foster

Homework 3

Due: **Monday, September 25, 2006.**

1. Which of the following functions are eigenfunctions of (a) $\frac{d}{dx}$, and (b) $\frac{d^2}{dx^2}$: (i) e^{ax} , (ii) e^{ax^2} , (iii) x , (iv) $ax + b$, (v) $\sin x$?
2. An extremely crude picture of an electron in an atom or molecule treats it as a particle in a one-dimensional box whose length is on the order of the size of atoms and molecules. (a) For an electron in a one-dimensional box of length 1.0\AA , calculate the separation between the two lowest energy levels. (b) Calculate the wavelength of a photon corresponding to a transition between these two levels. (c) In which portion of the electromagnetic spectrum is this wavelength?
3. McQuarrie, Chapter 3, # 18
4. McQuarrie, Chapter 3, # 22
5. McQuarrie, Chapter 3, # 23
6. McQuarrie, Chapter 3, # 26
7. The wavefunction of a harmonic oscillator in its ground state, as we will soon find out, is $\psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha x^2}{2}}$, with $\alpha = \left(\frac{k\mu}{\hbar^2}\right)^{\frac{1}{2}}$. Calculate the values of (a) $\langle x \rangle$, (b) $\langle x^2 \rangle$, (c) $\langle p_x \rangle$, (d) $\langle p_x^2 \rangle$. (e) Examine the value of $\Delta x \Delta p_x$ in the light of the uncertainty principle. Hint. Use the following integrals:
$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} \quad \int_0^{\infty} x e^{-\alpha x^2} dx = \frac{1}{2\alpha} \quad \int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx = \frac{1}{2} \left(\frac{\pi}{\alpha^3}\right)^{\frac{1}{2}}$$
8. The Laplace transform operator, \hat{L} , is defined by $\hat{L}f(x) = \int_0^{\infty} e^{-px} f(x) dx$.
 - a. Is \hat{L} linear?
 - b. Evaluate $\hat{L}(1)$.
 - c. Evaluate $\hat{L}e^{ax}$, assuming $p > a$
9. McQuarrie, Chapter 4, # 9
10. McQuarrie, Chapter 4, # 13