Error Analysis

Good statistics website
http://faculty.vassar.edu/lowry/webtext.html
see Salter paper for LS analysis

Propagation of errors

rules for error propagation pg56 Table 3.1 in Harris

addition or subtraction

$a + b + c = d \qquad a - b - c = d$

$s_d = (s_a^2 + s_b^2 + s_c^2)^{1/2}$

multiplication

$a*b/c = d$

$s_d/d = [(s_a/a)^2 + (s_b/b)^2 + (s_c/c)^2)^{1/2}]$

exponents

$y = x^a$

$s_y/y = a\, s_x/x$

logs

$y = \ln x$

$s_y = s_x/x$

$y = e^x$

$s_y/y = s_x$

Least squares analysis

The "best fit" line

Minimize the square of the vertical deviation - d_i^2

$$d_i^2 = (y_i - y)^2 = (y_i - (mx_i + b))^2$$

After some calculus….

Slope = $[n\Sigma(x_i y_i) - \Sigma x_i \Sigma y_i]/D$

Where D = $n\Sigma(x_i^2) - (\Sigma x_i)^2$

Intercept = $[\Sigma(x_i^2)\Sigma y_i - \Sigma x_i \Sigma(x_i y_i)]/D = y_{ave} - mx_{ave}$

Error analysis using Least Squares

Standard Error of Regression or standard error of residuals

The sum of the squares of the vertical deviates

$$s_y = [\Sigma(d_i - d_{av})^2/(N-2)]^{1/2} = [\Sigma(d_i)^2/(N-2)]^{1/2}$$
$$= [\Sigma(yi-mxi-b)^2/(N-2)]^{1/2}$$

error in slope = $s_m = [ns_y^2/D]^{1/2}$

error in intercept = $s_b = [s_y^2 \Sigma(x_i^2)/D]^{1/2}$

For typical calibration plot:

error in unknown x from interpolation

$$s_x = \{ (s_y/m)^2 \ [1/k_{unk} + 1/D (nx_{int}^2 - 2x_{int}\Sigma x_i + \Sigma(x_i^2))] \}^{1/2}$$

where k = number of replicate measurements of the unknown

For standard addition plot:

error in x intercept

$$s_x = \{ (s_y/m)^2 \ [1/D (nx_{int}^2 - 2x_{int}\Sigma x_i + \Sigma(x_i^2))] \}^{1/2}$$