Chem 116 POGIL Worksheet - Week 1 - Solutions
Gas Laws - Part 1

Key Questions

1. Using SI units of kilograms, meters, and seconds with these fundamental equations, determine the combination of units that define the following:
   i. The newton (N), the fundamental unit of force
   $$F = ma = (\text{kg})(\text{m·s}^{-2}) = \text{kg·m·s}^{-2} = \text{N}$$
   ii. The pascal (Pa), the fundamental unit of pressure
   $$P = F/A = \text{kg·m·s}^{-2}/\text{m}^2 = \text{N/m}^2 = \text{Pa}$$

2. The pressure of a column of liquid is given by $P = gdh$, where g is the acceleration of gravity (9.807 m·s^{-2}), d is the density, and h is the height. The density of mercury is 13.6 g/cm^3. What is the pressure in Pa exerted by a 760-mm column of mercury?
   $$P = (9.807\ \text{m·s}^{-2})(13.6\ \text{g·cm}^{-3})(10^6\ \text{cm}^3/\text{m}^3)(\text{kg}/10^3\ \text{g})(0.760\ \text{m}) = 1.01 \times 10^5\ \text{Pa}$$

3. In the United States we quote barometric pressure in inches of mercury (in. Hg). What is one standard atmosphere of pressure in these units? (1 in. ≡ 2.54 cm)
   $$1\ \text{atm} = (760\ \text{mm Hg})(\text{in. Hg}/25.4\ \text{mm Hg}) = 29.92\ \text{in. Hg}$$

4. In Europe barometric pressure is quoted in hectopascals (hPa), where *hecto* means one hundred. What is one standard atmosphere in hPa?
   $$1\ \text{atm} = (1.01 \times 10^5\ \text{Pa})(\text{hPa}/10^2\ \text{Pa}) = 1010\ \text{hPa}$$

5. A typical 10W-40 motor oil has a density of 0.875 g/cm^3. If you were to construct a barometer using 10W-40 motor oil, how high would the column be when the pressure is one atmosphere?
   $$h = (760\ \text{mm})(13.6\ \text{g/cm}^3/0.875\ \text{g/cm}^3) = 1.18 \times 10^4\ \text{mm} = 11.8\ \text{m}$$

6. Refer to the pictures of closed- and open-ended manometers above.
   i. In the manometer on the left, the left side reads 108 mm and the right side reads 32 mm. What is the pressure of the sample gas in torr?
   $$P_{\text{gas}} = 108\ \text{mm Hg} - 32\ \text{mm Hg} = 76\ \text{mm Hg} = 76\ \text{torr}$$

   ii. In the manometer on the right, the left (outer) side reads 98.3 mm, and the right (inner) side reads 32.1 mm. The pressure in the laboratory is 756.2 mm Hg. What is the pressure of the sample gas in torr?
   $$P_{\text{gas}} = (98.3\ \text{mm Hg} - 32.1\ \text{mm Hg}) + 756.2\ \text{mm Hg} = 66.2\ \text{mm Hg} + 756.2\ \text{mm Hg}$$
   $$= 822.4\ \text{mm Hg} = 822.4\ \text{torr}$$

   iii. In the manometer on the right, if the heights of the two sides were reversed (32.1 mm on the left and 98.3 mm on the right), what would the pressure of the sample gas be in torr?
   $$P_{\text{gas}} = 756.2\ \text{torr} - 66.2\ \text{torr} = 690.0\ \text{torr}$$

7. Carry out the following steps to derive equations for Boyle's Law.

   i. Express Boyle's Law as a proportion between V and P.

$$V \propto 1/P$$

   ii. A proportionality can be changed into an equation by adding a proportionality constant. For example, if $y \propto x$, we can use a constant, c, to write the equation $y = cx$. Do this for the Boyle's Law proportionality you just wrote, making b be the constant.

$$V = b/P \text{ or } VP = b$$

   iii. Under what conditions is b a constant?

A fixed amount of gas and constant temperature.

   iv. Consider a gas sample at constant temperature with an initial volume V_1 and an initial pressure P_1. Write an equation that shows how V_1 and P_1 are related to new volume and pressure conditions, V_2 and P_2, if either is changed. [Hint: look at the equation you wrote in part ii.]

$$V_1P_1 = V_2P_2 = b$$

8. A 144-mL sample of gas in a piston chamber has a pressure of 2.25 atm. If the piston is pushed in so that the gas has a volume of 36.0 mL, what is the new pressure? Assume the temperature remains constant.

$$P_2 = (2.25 \text{ atm})(144 \text{ mL}/36.0 \text{ mL}) = 9.00 \text{ atm}$$

9. Carry out the following steps to derive equations for Charles' Law.

   i. Write a proportionality between V and T that expresses Charles' Law.

$$V \propto T$$

   ii. Using the proportionality constant c, write an equation for the relationship between V and T.

$$V = cT \text{ or } V/T = c$$

   iii. Under what conditions is c a constant?

A fixed amount of gas at constant pressure.

   iv. What does "absolute temperature" mean? Why must we use absolute temperature in the equation for Charles' Law?

Absolute temperature means a temperature scale whose zero point is the lowest conceivable temperature; i.e., absolute zero. The kelvin scale (K) is an absolute temperature scale.

   v. Write an equation for the relationship between initial temperature and volume, T_1 and V_1, and final temperature and volume, T_2 and V_2, when either of these is changed under constant pressure conditions.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

10. A 5.00-L sample of gas in a piston chamber at 25 °C is heated to 100 °C, while the pressure is maintained at 1.00 atm. What is the volume of the gas after heating?

    T_1 = (273 + 25) K = 298 K T_2 = (273 + 100) K = 373 K

    V_2 = (5.00 L)(373 K/298 K) = 6.26 L

11. Carry out the following steps to derive equations for the Combined Gas Law.

    i. Combine your two proportionality expressions for Boyle's Law and for Charles' Law into one expression that shows how volume depends on both pressure and temperature.

    $$V \propto \frac{T}{P}$$

    ii. Using the proportionality constant *k*, write an equation for this combined gas law.

    $$V = \frac{kT}{P}$$

    iii. Write an equation for the relationship between initial conditions V_1, P_1, and T_1 and final conditions V_2, P_2, and T_2, when any two of these are changed for a sample of gas.

    $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

12. At 29 °C and 756 torr, a gas occupies 2.50 L. What is the volume if the temperature is changed to 159 °C, and the pressure is changed to 1550 torr?

    T_1 = 29 °C = (29 + 273) K = 302 K T_2 = 159 °C = (159 + 273) K = 432 K

    P_1 = 756 torr P_2 = 1550 torr

    V_1 = 2.50 L V_2 = ?

    $$V_2 = (2.50\ \text{L})\left(\frac{432\ \text{K}}{302\ \text{K}}\right)\left(\frac{756\ \text{torr}}{1550\ \text{torr}}\right) = 1.74\ \text{L}$$

13. A sample of He(*g*) occupies 15.0 L with a pressure of 856 torr when the temperature is 52 °C. How many grams of He(*g*) does the sample contain? [at. wt. He = 4.00 u]

    We will use *PV* = *nRT*, where *R* = 0.08206 L·atm/K·mol. Given the units of *R*, we must express the pressure in atmospheres (atm), the temperature in Kelvin, and the volume in liters. If we first solve for *n*, we can then calculate the number of grams from the atomic weight.

    $PV = nRT \Rightarrow n = PV/RT$ T = (273 + 52)K = 325 K

$$n = \frac{(856 \text{ torr}/760 \text{ torr}\cdot\text{atm}^{-1})(15.0 \text{ L})}{(0.08206 \text{ L}\cdot\text{atm/K}\cdot\text{mol})(325)\text{K}} = 0.633 \text{ mol He}$$

g He = (0.633 mol)(4.00 g/mol) = 2.53 g He

14. Derive the equation for Charles' Law by rearranging $PV = nRT$ to put all the constants on one side of the equation. Then, redefine the product of the constants to be the constant c.

    For Charles' Law, the variables are V and T, and the constants are n, P, and R.

    $$PV = nRT \Rightarrow V/T = nR/P = c$$

15. Amonton's Law expresses the relationship between temperature and pressure for a gas sample under constant volume. Derive the expression for Amonton's Law, using a as a constant. Then write the relationship between initial conditions T_1 and P_1 and final conditions T_2 and P_2, when amount and volume are constant.

    $$\frac{T}{P} = \frac{V}{nR} = a$$

    $$\frac{T_1}{P_1} = \frac{T_2}{P_2}$$

16. Avogadro's Law is based on the relationship between the amount of gas and its volume under constant temperature and pressure conditions. Derive an equation for Avogadro's Law from $PV = nRT$ to show the relationship between initial conditions n_1 and V_1 and final conditions n_2 and V_2.

    $V/n = RT/P = \text{const} \Rightarrow V_1/n_1 = V_2/n_2$ or $n_1/V_1 = n_2/V_2$

17. Avogadro's Law was an extension of an earlier observation of Gay-Lussac. Gay-Lussac's Law says that in reactions between gases at constant temperature and pressure, the volumes that react are in the ratios of small whole numbers. Nitrogen gas and hydrogen gas combine directly to form ammonia gas. Write the balanced equation for this reaction. If 250 mL of nitrogen gas at 5.00 atm and 750 K is reacted with hydrogen gas at the same pressure and temperature, what volume of hydrogen gas is required? What is the theoretical yield in milliliters of ammonia gas under these conditions?

    $$3\,H_2(g) + N_2(g) \rightarrow 2\,NH_3(g)$$

    $V(H_2)$ = (250 mL N_2)(3 mL H_2/1 mL N_2) = 750 mL H_2

    $V(NH_3)$ = (250 mL N_2)(2 mL NH_3/1 mL N_2) = 500 mL NH_3

18. What volume at STP does a 76.0-g sample of N_2 (m.w. = 28.0 u) occupy?

    V = (76.0 g)(mol/28.0 g)(22.4 L/mol) = 60.8 L

19. A gas sample with a mass of 57.2 g at STP occupies 80.0 L. Which of the following gases might it be: N_2, CO_2, CH_4?

n = (80.0 L)(1 mol/22.4 L) = 3.57 mol

m.w. = 57.2 g/3.57 mol = 16.0 g/mol

m.w. N_2 = 28.0 g/mol; m.w. CO_2 = 44.0 g/mol; m.w. CH_4 = 16.0 g/mol $\Rightarrow$ CH_4